Internet Appendix to

Reassessing False Discoveries in Mutual Fund Performance: Skill, Luck, or Lack of Power?

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A. Student-t Nonzero alphas

In this section, we present results on the sensitivity of the False Discovery Rate (FDR) estimators to the presence of fat tails.

For this purpose, we simulate fund alphas from a mixture of a point mass at 0 and a *t*-distribution, that is,

$$\alpha \sim \pi^0 \delta^0 + (\pi^- + \pi^+) f_{t(\nu_\alpha)},$$
 (IA.1)

varying across simulations the proportion of zero-alpha funds (π^0) from 95% to 1% and the degrees of freedom of the t-distribution (ν_α) from 2 to 25. In Table IA.I, we present the results from this simulation analysis. The main effect we observe in this table is one we also note in Section II.B in the paper: the bias in the FDR estimates increases as the point mass at zero becomes smaller (as we move down across the table). The secondary effect is that the presence of fatter tails in the fund alpha distribution (as we move to the left across the table) ameliorates the bias in the FDR estimates. However, this effect is very weak and the bias is still large even when the tails are very fat.

Table IA.I: Estimates of Alpha Group Proportions — Student-t Nonzero αs

Results from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of Student's t distributions. That is, $\alpha \sim \pi^0 \delta^0 + (\pi^- + \pi^+) f_{t(\nu_\alpha)}$, where π^0 , π^- , and π^+ are the proportions of funds with zero, negative, and positive α , respectively, δ^0 is the Dirac mass at 0, and $f_{t(\nu_\alpha)}$ is the density of the Student-t with ν_α degrees of freedom. t (ν_α) is symmetric around 0, so $\pi^- = \pi^+$. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the degrees of freedom ν_α of the t distribution for the nonzero αs (written in the table's top row). Other simulation parameters are as described in Section II.A in the paper. In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias.

	$v_{\alpha}=2$	$v_{\alpha}=3$	$v_{\alpha} = 5$	$v_{\alpha} = 10$	$v_{\alpha} = 15$	$v_{\alpha}=25$
DGP A-T-1: $\pi^0 = 95.00\%$	97.84 (2.95)	98.14 (2.95)	98.45 (2.95)	98.34 (2.95)	98.36 (2.95)	98.29 (2.95)
$\pi^- = 2.50\%$	0.89 (2.63)	0.75 (2.63)	0.68 (2.63)	0.70 (2.63)	0.68 (2.63)	0.70 (2.63)
$\pi^{+} = 2.50\%$	1.26 (1.33)	1.11 (1.32)	0.87 (1.32)	0.97 (1.32)	0.97 (1.32)	1.01 (1.33)
DGP A-T-2: $\pi^0 = 75.00\%$	92.71 (2.93)	93.99 (2.93)	94.86 (2.94)	95.04 (2.94)	95.06 (2.94)	95.19 (2.94)
$\pi^- = 12.50\%$	3.41 (2.62)	2.77 (2.63)	2.29 (2.63)	2.19 (2.63)	2.12 (2.63)	2.14 (2.63)
$\pi^+ = 12.50\%$	3.87 (1.36)	3.24 (1.35)	2.85 (1.35)	2.77 (1.34)	2.82 (1.34)	2.68 (1.34)
DGP A-T-3: $\pi^0 = 50.00\%$	85.84 (2.89)	87.62 (2.90)	89.12 (2.91)	90.57 (2.92)	90.59 (2.92)	90.98 (2.92)
$\pi^- = 25.00\%$	6.75 (2.61)	5.71 (2.62)	5.00 (2.62)	4.45 (2.62)	4.34 (2.62)	4.22 (2.62)
$\pi^+ = 25.00\%$	7.41 (1.39)	6.67 (1.38)	5.89 (1.37)	4.98 (1.37)	5.07 (1.36)	4.80 (1.36)
DGP A-T-4: $\pi^0 = 20.00\%$	77.29 (2.83)	80.30 (2.85)	82.48 (2.87)	84.65 (2.88)	85.05 (2.89)	85.83 (2.89)
$\pi^- = 40.00\%$	10.70 (2.58)	9.25 (2.60)	8.07 (2.60)	7.13 (2.61)	6.84 (2.61)	6.67 (2.61)
$\pi^+ = 40.00\%$	12.01 (1.42)	10.44 (1.41)	9.45 (1.40)	8.22 (1.39)	8.11 (1.39)	7.50 (1.39)
DGP A-T-5: $\pi^0 = 10.00\%$	74.98 (2.81)	78.21 (2.84)	80.45 (2.85)	82.60 (2.87)	83.43 (2.87)	83.53 (2.88)
$\pi^- = 45.00\%$	11.91 (2.57)	10.26 (2.59)	9.00 (2.60)	8.08 (2.60)	7.70 (2.61)	7.47 (2.61)
$\pi^+ = 45.00\%$	13.11 (1.43)	11.53 (1.42)	10.55 (1.41)	9.33 (1.40)	8.86 (1.40)	9.00 (1.40)
DGP A-T-6: $\pi^0 = 5.00\%$	73.25 (2.79)	76.70 (2.82)	79.39 (2.84)	81.75 (2.86)	82.38 (2.87)	82.68 (2.87)
$\pi^- = 47.50\%$	12.53 (2.57)	11.04 (2.58)	9.59 (2.59)	8.45 (2.60)	8.10 (2.60)	8.02 (2.60)
$\pi^+ = 47.50\%$	14.22 (1.44)	12.25 (1.42)	11.01 (1.41)	9.80 (1.40)	9.52 (1.40)	9.31 (1.40)
DGP A-T-7: $\pi^0 = 2.00\%$	72.81 (2.79)	76.49 (2.82)	78.95 (2.84)	80.87 (2.86)	82.20 (2.87)	82.34 (2.87)
$\pi^- = 49.00\%$	13.00 (2.56)	11.17 (2.58)	9.76 (2.59)	8.84 (2.60)	8.31 (2.60)	8.03 (2.60)
$\pi^+ = 49.00\%$	14.19 (1.44)	12.34 (1.43)	11.29 (1.42)	10.30 (1.41)	9.49 (1.40)	9.62 (1.40)
DGP A-T-8: $\pi^0 = 1.00\%$	72.26 (2.78)	75.85 (2.81)	78.79 (2.84)	80.92 (2.86)	81.63 (2.86)	82.29 (2.87)
$\pi^- = 49.50\%$	13.18 (2.56)	11.30 (2.58)	9.92 (2.59)	8.77 (2.60)	8.53 (2.60)	8.26 (2.60)
$\pi^+ = 49.50\%$	14.56 (1.44)	12.84 (1.43)	11.28 (1.42)	10.31 (1.41)	9.84 (1.40)	9.45 (1.40)

B. Cross-sectional Error Correlation

In this section, we present results on the sensitivity of the FDR estimators to the presence of cross-sectional correlation in the error term of the model of returns. In particular, we focus on correlation arising form a latent linear factor structure in the errors as in Jones and Shanken (2005), which captures the role of non-priced factors. We generate unbalanced panels with cross-sectional and time-series dimensions similar to those in the real data and we use the baseline discrete data generating processes (DGPs) for alpha, but we replace the baseline assumption $\varepsilon_{it} \sim \mathcal{N}\left(0, \sigma^2\right)$ in the paper with $\varepsilon_{it} := G_t' \delta_i + \xi_{it}$. G_t is a vector of four latent factors, one on which all funds may load and three strategy-specific factors on which only funds with a specific investment strategy (Growth & Income, Growth, or Aggressive Growth) may load, δ_i are fund-specific loadings, and $\xi_{it} \sim \mathcal{N}\left(0, \sigma_i^{*2}\right)$ with σ_i^* a fund-specific standard deviation. The fund-specific parameters δ_i and σ_i^* we use in our simulations are estimated from the data (for details on the estimation, see Geweke and Zhou (1996) and Andrikogiannopoulou and Papakonstantinou (2016)).

We note that the baseline assumption $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma^2\right)$ assumes error independence and also error variance homogeneity across funds, while $\varepsilon_{it} := G'_t \delta_i + \xi_{it}$ not only allows for cross-sectional error correlation but also for error variance heterogeneity. As a result, we first present in Table IA.II intermediate results on simulations that simply replace the assumption $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma^2\right)$ with $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma_i^2\right)$, where σ_i^2 is fund-specific error variance. That is, Table IA.II is a variation of Table V in the paper, the only difference being that the former allows for error variance heterogeneity.

Next, in Table IA.III, we present the results from the simulation analysis that allows for both error variance heterogeneity and cross-sectional error correlation. Comparing these results with those in Table IA.II, we see that the mean and standard deviation of the FDR estimates is very similar both with (in Table IA.III) and without (in Table IA.II) cross-sectional error correlation, for most alpha DGPs. The main difference is that — as a consequence of the increased estimator variability in the presence of cross-sectional error correlation that is discussed in detail in Section II.D in the paper (and which is not taken into account by the *analytic* standard deviations reported in Tables IA.III and IA.II) — when the true proportions are near the natural boundaries of 0 and 1, the FDR estimator becomes biased, even for very large values of $\bar{\alpha}$. Indeed, in Table IA.III, we see that for the first DGP (for which $\pi^+ = 0.5\%$) the estimates of π^+ are five to seven times larger than the true value, even for $\bar{\alpha} = 3.5\%$.

Table IA.II: Estimates of Alpha Group Proportions — Heterogeneous Errors

Results from simulations in which — as in Table V in the paper — the sample of funds is an unbalanced panel and nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions (i.e., $\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+$), but additionally the variance of the error in the model of returns is heterogeneous across funds. That is, in Equation 3 we replace the assumption $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma^2\right)$ with $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma_i^2\right)$. The distribution of σ_i used in the simulations is estimated from the residuals of the fund-by-fund regressions using the real data; the mean estimated value across funds is 0.022. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias.

	$\bar{\alpha} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{a}=2.5$	$\bar{a} = 3.0$	$\bar{a} = 3.5$
DGP <i>A-D-1</i> : $\pi^0 = 93.75\%$	98.48 (2.47)	97.80 (2.47)	97.33 (2.47)	96.57 (2.46)	96.15 (2.46)	95.76 (2.46)
$\pi^- = 5.75\%$	0.90 (2.19)	1.49 (2.19)	2.02 (2.19)	2.56 (2.19)	3.05 (2.19)	3.37 (2.19)
$\pi^{+} = 0.50\%$	0.62 (1.09)	0.71 (1.08)	0.66 (1.08)	0.87 (1.08)	0.80 (1.08)	0.87 (1.08)
DGP A-D-2: $\pi^0 = 87.50\%$	97.98 (2.47)	96.27 (2.46)	95.04 (2.46)	93.57 (2.45)	92.08 (2.45)	91.14 (2.44)
$\pi^- = 11.50\%$	1.54 (2.19)	3.07 (2.18)	4.34 (2.18)	5.62 (2.18)	6.87 (2.17)	7.72 (2.17)
$\pi^+ = 1.00\%$	0.49 (1.08)	0.66 (1.07)	0.62 (1.07)	0.81 (1.06)	1.06 (1.06)	1.13 (1.06)
DGP <i>A-D-3</i> : $\pi^0 = 75.00\%$	96.39 (2.46)	93.11 (2.45)	89.74 (2.44)	87.03 (2.42)	84.94 (2.41)	82.69 (2.40)
$\pi^- = 23.00\%$	3.35 (2.18)	6.48 (2.17)	9.51 (2.16)	12.04 (2.15)	14.02 (2.14)	15.93 (2.13)
$\pi^{+} = 2.00\%$	0.27 (1.05)	0.42 (1.04)	0.75 (1.03)	0.93 (1.02)	1.04 (1.02)	1.38 (1.02)
DGP A-D-4: $\pi^0 = 62.50\%$	94.12 (2.45)	89.43 (2.44)	85.01 (2.41)	80.61 (2.39)	77.23 (2.37)	74.09 (2.34)
$\pi^- = 34.50\%$	5.66 (2.17)	10.15 (2.15)	14.41 (2.13)	18.45 (2.11)	21.45 (2.09)	24.19 (2.07)
$\pi^{+} = 3.00\%$	0.22 (1.03)	0.42 (1.00)	0.58 (0.99)	0.94 (0.98)	1.32 (0.97)	1.71 (0.97)
DGP <i>A-D-5</i> : $\pi^0 = 50.00\%$	92.58 (2.45)	85.96 (2.42)	79.73 (2.38)	74.07 (2.34)	69.30 (2.30)	65.66 (2.27)
$\pi^- = 46.00\%$	7.32 (2.15)	13.79 (2.12)	19.64 (2.09)	24.76 (2.06)	28.92 (2.03)	32.18 (2.00)
$\pi^{+} = 4.00\%$	0.10 (1.00)	0.25 (0.96)	0.63 (0.95)	1.17 (0.93)	1.77 (0.92)	2.16 (0.92)
DGP <i>A-D-6</i> : $\pi^0 = 37.50\%$	90.70 (2.44)	82.75 (2.40)	74.78 (2.35)	67.55 (2.28)	61.69 (2.22)	57.17 (2.17)
$\pi^- = 57.50\%$	9.25 (2.14)	17.03 (2.10)	24.71 (2.06)	31.24 (2.01)	36.39 (1.96)	40.17 (1.92)
$\pi^{+} = 5.00\%$	0.05 (0.97)	0.21 (0.93)	0.51 (0.90)	1.22 (0.89)	1.92 (0.87)	2.66 (0.86)
DGP A-D-7: $\pi^0 = 25.00\%$	88.80 (2.43)	78.77 (2.38)	69.54 (2.30)	61.35 (2.22)	54.06 (2.13)	48.41 (2.05)
$\pi^- = 69.00\%$	11.16 (2.13)	21.13 (2.07)	29.94 (2.01)	37.37 (1.94)	43.49 (1.88)	48.41 (1.81)
$\pi^{+} = 6.00\%$	0.03 (0.94)	0.11 (0.89)	0.53 (0.86)	1.29 (0.83)	2.45 (0.82)	3.18 (0.81)
DGP A-D-8: $\pi^0 = 6.25\%$	85.75 (2.42)	74.12 (2.34)	61.68 (2.22)	51.31 (2.09)	42.70 (1.96)	35.19 (1.81)
$\pi^- = 86.25\%$	14.23 (2.10)	25.85 (2.03)	37.79 (1.93)	47.01 (1.83)	54.44 (1.72)	60.75 (1.61)
$\pi^{+} = 7.50\%$	0.02 (0.89)	0.03 (0.83)	0.54 (0.78)	1.69 (0.75)	2.86 (0.73)	4.06 (0.71)

Table IA.III: Estimates of Alpha Group Proportions — Heterogeneous and Correlated Errors

Results from simulations in which the sample of funds is an unbalanced panel and nonzero α s (expressed as annualized percentages) are generated from a variety of discrete distributions (i.e., $\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+$ as in Table V), but additionally the errors in the model of returns are heterogeneous and correlated across funds. That is, we allow for a heterogeneous latent linear factor structure for the error as in Jones and Shanken (2005), i.e., in Equation 3 we replace the assumption $\varepsilon_{it} \sim \mathcal{N}\left(0,\sigma^2\right)$ with $\varepsilon_{it} = G_t'\delta_t + \xi_{it}$, where G_t is the vector of latent error factors, δ_i are fund-specific error factor loadings, and $\xi_{it} \sim \mathcal{N}\left(0,\sigma_i^{*2}\right)$ with σ_i^{*2} fund-specific. The fund-specific δ_i and σ_i^* used in the simulations are estimated from the data. Factor returns and loadings are drawn from their empirical distributions, as described in Section II.A in the paper. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero α s from zero (written in the table's top row). In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to more biased estimates.

	$\bar{\alpha} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{a}=2.5$	$\bar{a} = 3.0$	$\bar{a} = 3.5$
DGP A-D-1: $\pi^0 = 93.75\%$	93.70 (2.45)	93.64 (2.45)	93.99 (2.45)	93.43 (2.45)	92.80 (2.45)	93.04 (2.45)
$\pi^- = 5.75\%$	3.83 (2.16)	3.24 (2.17)	3.10 (2.17)	3.85 (2.17)	3.67 (2.17)	3.55 (2.17)
$\pi^{+} = 0.50\%$	2.47 (1.04)	3.12 (1.06)	2.91 (1.05)	2.72 (1.05)	3.53 (1.06)	3.42 (1.06)
DGP <i>A-D-2</i> : $\pi^0 = 87.50\%$	93.85 (2.45)	93.69 (2.45)	92.95 (2.45)	91.54 (2.44)	92.62 (2.44)	91.25 (2.44)
$\pi^- = 11.50\%$	3.78 (2.17)	3.47 (2.17)	4.40 (2.16)	5.48 (2.16)	4.19 (2.16)	4.85 (2.16)
$\pi^+ = 1.00\%$	2.36 (1.04)	2.85 (1.04)	2.66 (1.04)	2.98 (1.03)	3.19 (1.03)	3.90 (1.05)
DGP <i>A-D-3</i> : $\pi^0 = 75.00\%$	93.75 (2.45)	92.70 (2.44)	90.75 (2.44)	87.61 (2.42)	84.87 (2.40)	82.37 (2.39)
$\pi^- = 23.00\%$	4.71 (2.15)	5.25 (2.15)	6.37 (2.15)	9.31 (2.14)	11.48 (2.12)	13.39 (2.11)
$\pi^{+} = 2.00\%$	1.54 (0.98)	2.05 (1.00)	2.88 (1.02)	3.08 (1.00)	3.65 (0.99)	4.24 (1.00)
DGP A-D-4: $\pi^0 = 62.50\%$	92.32 (2.44)	88.73 (2.42)	85.54 (2.41)	81.59 (2.38)	77.66 (2.36)	74.59 (2.34)
$\pi^- = 34.50\%$	5.60 (2.15)	8.96 (2.13)	11.94 (2.12)	16.37 (2.09)	18.97 (2.08)	22.23 (2.06)
$\pi^{+} = 3.00\%$	2.07 (0.99)	2.32 (0.98)	2.52 (0.97)	2.04 (0.94)	3.36 (0.95)	3.18 (0.95)
DGP <i>A-D-5</i> : $\pi^0 = 50.00\%$	90.41 (2.43)	86.02 (2.41)	80.94 (2.38)	74.07 (2.33)	69.71 (2.30)	66.61 (2.27)
$\pi^- = 46.00\%$	7.32 (2.13)	11.92 (2.11)	16.96 (2.09)	23.35 (2.05)	26.97 (2.03)	29.82 (2.01)
$\pi^{+} = 4.00\%$	2.26 (0.97)	2.06 (0.94)	2.10 (0.93)	2.59 (0.92)	3.33 (0.92)	3.56 (0.92)
DGP <i>A-D-6</i> : $\pi^0 = 37.50\%$	90.87 (2.44)	83.57 (2.39)	77.30 (2.35)	69.32 (2.29)	63.44 (2.23)	58.15 (2.18)
$\pi^- = 57.50\%$	7.57 (2.13)	14.90 (2.09)	20.87 (2.06)	28.86 (2.01)	34.20 (1.96)	39.42 (1.92)
$\pi^{+} = 5.00\%$	1.56 (0.95)	1.53 (0.91)	1.83 (0.90)	1.82 (0.87)	2.36 (0.86)	2.42 (0.84)
DGP <i>A-D-7</i> : $\pi^0 = 25.00\%$	87.38 (2.42)	81.99 (2.38)	71.60 (2.31)	63.81 (2.23)	54.86 (2.13)	49.99 (2.06)
$\pi^- = 69.00\%$	10.95 (2.11)	17.07 (2.08)	27.21 (2.01)	34.87 (1.95)	42.98 (1.87)	46.84 (1.82)
$\pi^{+} = 6.00\%$	1.67 (0.91)	0.95 (0.89)	1.19 (0.84)	1.32 (0.83)	2.16 (0.80)	3.18 (0.80)
DGP A-D-8: $\pi^0 = 6.25\%$	87.79 (2.41)	77.07 (2.35)	62.76 (2.22)	52.63 (2.09)	44.82 (1.97)	37.43 (1.84)
$\pi^- = 86.25\%$	11.42 (2.09)	22.18 (2.03)	36.31 (1.92)	45.79 (1.82)	52.51 (1.73)	58.86 (1.62)
$\pi^{+} = 7.50\%$	0.79 (0.87)	0.76 (0.82)	0.93 (0.77)	1.58 (0.74)	2.67 (0.73)	3.70 (0.71)

C. Variations of DGPs

In this section, we present simulation results from variations of the data generating processes we use in the simulations in the paper.

Table IA.IV is a variation of Table I in the paper, the difference being that in the former we simulate alphas from a distribution in which the ratio $\frac{\pi^-}{\pi^+}$ of negative-alpha to positive-alpha funds is smaller. In more detail, in the simulations in both tables, nonzero α s are generated from a variety of discrete distributions ($\alpha \sim \pi^0 \delta^0 + \pi^- \delta^-_{\bar{\alpha}} + \pi^+ \delta^+_{\bar{\alpha}}$), but in Table I in the paper the ratio $\frac{\pi^-}{\pi^+}$ equals 11.5 while in Table IA.IV this ratio equals 6. The ratio $\frac{\pi^-}{\pi^+} = 11.5$ is the one that Barras, Scaillet and Wermers (2010) — hereafter BSW — use in their simulations, therefore we also use it for comparability in Table I and in most of our analysis in the paper. The ratio $\frac{\pi^-}{\pi^+} = 6$ is the one estimated by Andrikogiannopoulou and Papakonstantinou (2016) using the real data.

Table IA.V is a variation of Table III in the paper, the difference being that in the former we simulate alphas from a normal distribution that is not centered at zero alpha. In more detail, in the simulations in both tables, nonzero α s are generated from a variety of normal distributions $(\alpha \sim \pi^0 \delta^0 + (\pi^- + \pi^+) f_{\mathcal{N}(\mu_\alpha, \sigma_\alpha^2)})$, but in Table III in the paper the normals are centered at $\alpha = 0$ while in Table IA.V the normals are centered at $\alpha = -0.8\%$ annualized. In Table III in the paper we use normals centered at $\alpha = 0$ for easier exposition, but here in Table IA.V we provide additional results from normals centered at $\alpha = -0.8\%$ as they are more realistic (see Jones and Shanken (2005), Andrikogiannopoulou and Papakonstantinou (2016)).

Table IA.VI is a variation of Table IV in the paper, the difference being that in the former we simulate alphas from a distribution in which nonzero alphas are larger. In more detail, in the simulations in both tables, nonzero α s are generated from $\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+$ with $\pi^0 = 9\%$, $\pi^- = 78\%$, $\pi^+ = 13\%$, but in Table IV in the paper the nonzero alphas are intermediate at $\bar{\alpha} = 1\%$ annualized while in Table IA.VI the nonzero alphas are large at $\bar{\alpha} = 2\%$ annualized. Intermediate nonzero alphas of about -1% and +1% (for negatives and positives, respectively) are roughly consistent with the estimation results of Andrikogiannopoulou and Papakonstantinou (2016) using the real data, hence we use these values in the analysis in Table IV in the paper. We use larger nonzero alphas of about -2% and +2% in Table IA.VI here, to examine whether our results on varying the number of funds N and the number of observations per fund T are sensitive to the magnitude of alpha.

Table IA.IV: Estimates of Alpha Group Proportions — Discrete Nonzero α s, with Small $\frac{\pi}{\pi^+}$

Results from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions. That is, $\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+$, where π^0 , π^- , π^+ are the proportions of funds with zero, negative, and positive α , respectively, and δ^0 , $\delta_{\bar{\alpha}}^-$, $\delta_{\bar{\alpha}}^+$ are Dirac masses at $0, -\bar{\alpha}, +\bar{\alpha}$, respectively. This analysis is similar to that presented in Table I in the paper, the difference being that the ratio of negative-alpha to positive-alpha funds is large ($\frac{\pi^-}{\pi^+} = 11.5$) in Table I and substantially smaller ($\frac{\pi^-}{\pi^+} = 6$) here. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ , and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero. In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias.

	$\bar{a} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{\alpha} = 2.5$	$\bar{\alpha} = 3.0$	$\bar{\alpha} = 3.5$
DGP <i>A-D-1b</i> : $\pi^0 = 96.50\%$	98.33 (2.95)	98.12 (2.95)	97.49 (2.95)	96.87 (2.94)	96.46 (2.94)	96.71 (2.94)
$\pi^- = 3.00\%$	0.80 (2.62)	1.06 (2.62)	1.47 (2.62)	1.96 (2.62)	2.43 (2.62)	2.28 (2.62)
$\pi^{+} = 0.50\%$	0.86 (1.31)	0.82 (1.31)	1.04 (1.31)	1.18 (1.31)	1.11 (1.31)	1.01 (1.31)
DGP A - D - $2b$: $\pi^0 = 93.00\%$	97.85 (2.95)	96.84 (2.94)	95.69 (2.94)	94.55 (2.94)	93.95 (2.93)	93.31 (2.93)
$\pi^- = 6.00\%$	1.34 (2.62)	2.11 (2.62)	3.16 (2.61)	4.03 (2.61)	4.74 (2.61)	5.28 (2.61)
$\pi^+ = 1.00\%$	0.81 (1.30)	1.05 (1.30)	1.14 (1.30)	1.42 (1.30)	1.31 (1.29)	1.41 (1.29)
DGP A - D - $3b$: $\pi^0 = 86.00\%$	97.14 (2.95)	94.04 (2.93)	91.68 (2.92)	89.08 (2.91)	87.59 (2.90)	86.74 (2.90)
$\pi^- = 12.00\%$	2.23 (2.61)	4.89 (2.60)	6.93 (2.60)	8.86 (2.59)	10.20 (2.58)	11.02 (2.58)
$\pi^{+} = 2.00\%$	0.63 (1.28)	1.08 (1.28)	1.39 (1.27)	2.06 (1.27)	2.21 (1.27)	2.24 (1.28)
DGP A - D - $4b$: $\pi^0 = 79.00\%$	95.70 (2.94)	91.65 (2.92)	87.67 (2.90)	83.98 (2.88)	81.63 (2.86)	80.20 (2.85)
$\pi^- = 18.00\%$	3.63 (2.60)	7.25 (2.59)	10.64 (2.58)	13.39 (2.56)	15.58 (2.55)	16.71 (2.54)
$\pi^{+} = 3.00\%$	0.68 (1.26)	1.11 (1.26)	1.69 (1.25)	2.62 (1.26)	2.79 (1.25)	3.09 (1.25)
DGP A - D - $5b$: $\pi^0 = 72.00\%$	94.29 (2.93)	88.54 (2.91)	83.05 (2.87)	78.41 (2.84)	75.35 (2.81)	73.75 (2.80)
$\pi^- = 24.00\%$	5.06 (2.60)	10.05 (2.57)	14.63 (2.55)	18.22 (2.53)	20.83 (2.51)	22.28 (2.50)
$\pi^{+} = 4.00\%$	0.65 (1.25)	1.42 (1.24)	2.32 (1.23)	3.38 (1.23)	3.82 (1.23)	3.97 (1.23)
DGP <i>A-D-6b</i> : $\pi^0 = 65.00\%$	92.48 (2.93)	85.84 (2.89)	78.83 (2.84)	73.20 (2.79)	69.21 (2.75)	66.92 (2.72)
$\pi^- = 30.00\%$	6.79 (2.59)	12.89 (2.56)	18.35 (2.52)	22.76 (2.49)	26.16 (2.46)	28.09 (2.44)
$\pi^{+} = 5.00\%$	0.72 (1.23)	1.27 (1.22)	2.82 (1.21)	4.04 (1.21)	4.63 (1.21)	4.99 (1.21)
DGP A-D-7b: $\pi^0 = 58.00\%$	91.46 (2.92)	82.99 (2.87)	74.42 (2.80)	68.07 (2.74)	63.42 (2.68)	60.51 (2.64)
$\pi^- = 36.00\%$	8.08 (2.58)	15.55 (2.54)	22.00 (2.49)	27.36 (2.44)	31.08 (2.41)	33.49 (2.38)
$\pi^{+} = 6.00\%$	0.46 (1.21)	1.45 (1.19)	3.57 (1.19)	4.57 (1.18)	5.50 (1.18)	6.00 (1.19)
DGP A - D - $8b$: $\pi^0 = 47.50\%$	88.87 (2.91)	78.89 (2.84)	68.06 (2.74)	59.61 (2.63)	54.07 (2.55)	50.62 (2.49)
$\pi^- = 45.00\%$	10.59 (2.56)	19.49 (2.51)	27.88 (2.43)	34.32 (2.36)	38.86 (2.30)	41.94 (2.26)
$\pi^{+} = 7.50\%$	0.54 (1.19)	1.62 (1.16)	4.06 (1.15)	6.08 (1.14)	7.07 (1.15)	7.44 (1.15)

Table IA.V: Estimates of Alpha Group Proportions — Normal Nonzero α s, Centered at -0.8%

Results from simulations with nonzero α s (expressed as annualized percentages) generated from a variety of normals. That is, $\alpha \sim \pi^0 \delta^0 + (\pi^- + \pi^+) f_{\mathcal{N}(-0.8,\sigma_\alpha^2)}$, where π^0 , π^- , π^+ are the proportions of funds with zero, negative, and positive α , δ^0 is the Dirac mass at 0, and $f_{\mathcal{N}(-0.8,\sigma_\alpha^2)}$ is the normal density with mean -0.8 and standard deviation σ_α . This analysis is similar to that presented in Table III in the paper, with the difference that here $\mathcal{N}\left(-0.8,\sigma_\alpha^2\right)$ is *not* symmetric around 0, so $\frac{\pi^-}{\pi^+}$ varies with σ_α . Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the standard deviation σ_α hence also in the ratio $\frac{\pi^-}{\pi^+}$ (both written in the table's top row). In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias; estimates with no bias have no shading and estimates with the maximum possible bias have the darkest shading.

	$\sigma_{\alpha} = 0.5$	$\sigma_{\alpha} = 1.0$	$\sigma_{\alpha} = 1.5$	$\sigma_{\alpha} = 2.0$	$\sigma_{\alpha} = 3.0$	$\sigma_{\alpha} = 5.0$
	$\frac{\pi^{-}}{\pi^{+}} = 17.3$	$\frac{\pi^{-}}{\pi^{+}} = 3.7$	$\frac{\pi^{-}}{\pi^{+}} = 2.4$	$\frac{\pi^{-}}{\pi^{+}} = 1.9$	$\frac{\pi^{-}}{\pi^{+}} = 1.5$	$\frac{\pi^{-}}{\pi^{+}} = 1.3$
DGP A-N-1b: $\pi^0 = 95.00\%$	98.31 (2.95)	98.03 (2.95)	97.65 (2.95)	97.53 (2.95)	97.13 (2.95)	96.30 (2.94)
	1.03 (2.62)	1.06 (2.62)	1.39 (2.62)	1.30 (2.62)	1.54 (2.62)	1.96 (2.63)
	0.66 (1.30)	0.91 (1.31)	0.96 (1.31)	1.17 (1.31)	1.33 (1.32)	1.75 (1.33)
DGP $A-N-2b$: $\pi^0 = 75.00\%$	95.70 (2.94)	93.36 (2.93)	91.17 (2.92)	88.85 (2.91)	85.68 (2.89)	81.67 (2.86)
	3.99 (2.60)	5.78 (2.60)	6.91 (2.60)	8.04 (2.60)	9.16 (2.60)	10.39 (2.59)
	0.30 (1.24)	0.86 (1.26)	1.92 (1.29)	3.11 (1.31)	5.16 (1.35)	7.93 (1.38)
DGP A-N-3b: $\pi^0 = 50.00\%$	91.46 (2.92)	87.66 (2.90)	82.52 (2.87)	78.09 (2.83)	70.91 (2.77)	63.90 (2.69)
	8.43 (2.56)	11.68 (2.56)	14.34 (2.56)	16.00 (2.55)	18.38 (2.53)	20.87 (2.49)
	0.11 (1.16)	0.66 (1.20)	3.15 (1.26)	5.91 (1.30)	10.71 (1.36)	15.23 (1.41)
DGP A-N-4b: $\pi^0 = 20.00\%$	86.35 (2.89)	80.01 (2.85)	72.25 (2.78)	64.90 (2.70)	53.98 (2.55)	42.28 (2.33)
	13.63 (2.51)	19.31 (2.50)	23.15 (2.48)	25.81 (2.45)	29.20 (2.38)	33.15 (2.27)
	0.02 (1.04)	0.68 (1.12)	4.60 (1.21)	9.29 (1.28)	16.82 (1.36)	24.57 (1.42)
DGP A-N-5b: $\pi^0 = 10.00\%$	84.82 (2.88)	77.24 (2.83)	68.56 (2.74)	60.54 (2.64)	48.51 (2.45)	35.10 (2.16)
	15.18 (2.49)	22.12 (2.48)	26.24 (2.45)	28.99 (2.41)	32.80 (2.32)	37.10 (2.17)
	0.00 (1.01)	0.64 (1.10)	5.20 (1.19)	10.47 (1.27)	18.69 (1.36)	27.80 (1.41)
DGP A-N-6b: $\pi^0 = 5.00\%$	83.55 (2.88)	75.71 (2.81)	67.00 (2.72)	58.09 (2.61)	45.81 (2.40)	31.43 (2.07)
	16.45 (2.48)	23.53 (2.46)	27.48 (2.44)	30.62 (2.39)	34.61 (2.29)	39.37 (2.11)
	0.00 (0.98)	0.76 (1.08)	5.53 (1.19)	11.29 (1.27)	19.58 (1.36)	29.20 (1.41)
DGP $A-N-7b$: $\pi^0 = 2.00\%$	83.33 (2.87)	75.17 (2.81)	65.54 (2.71)	57.35 (2.60)	43.85 (2.36)	29.45 (2.01)
	16.66 (2.47)	24.16 (2.46)	28.62 (2.42)	31.42 (2.38)	35.86 (2.27)	40.42 (2.08)
	0.01 (0.97)	0.67 (1.07)	5.85 (1.18)	11.23 (1.26)	20.29 (1.36)	30.13 (1.41)
DGP A-N-8b: $\pi^0 = 1.00\%$	83.08 (2.87)	74.93 (2.81)	65.29 (2.70)	56.95 (2.59)	43.63 (2.36)	28.77 (1.99)
	16.91 (2.47)	24.43 (2.46)	28.84 (2.42)	31.68 (2.37)	36.07 (2.26)	40.94 (2.07)
	0.00 (0.96)	0.65 (1.07)	5.87 (1.18)	11.37 (1.26)	20.31 (1.35)	30.29 (1.40)

Table IA.VI: Estimates of Alpha Group Proportions — Varying N and T, with Large Nonzero α s

Results from simulations in which we vary the number of funds N and the number of observations T for each fund in the sample, assuming a balanced panel. In all simulations, αs (expressed as annualized percentages) are generated from $\alpha \sim \pi^0 \delta^0 + \pi^- \delta^-_{\bar{\alpha}} + \pi^+ \delta^+_{\bar{\alpha}}$, with $\pi^0 = 9\%$, $\pi^- = 78\%$, $\pi^+ = 13\%$ the proportions of funds with zero, negative, and positive α , respectively, and δ^0 , $\delta^-_{\bar{\alpha}}$, $\delta^+_{\bar{\alpha}}$ the Dirac masses at $0, -\bar{\alpha}, +\bar{\alpha}$, respectively. This analysis is similar to that presented in Table IV in the paper, with the difference that here we have $\bar{\alpha} = 2\%$ instead of $\bar{\alpha} = 1\%$. The values for π^0 , π^- , π^+ are motivated by the estimation results of Andrikogiannopoulou and Papakonstantinou (2016) using the real data for U.S. equity mutual funds. Across rows, the DGPs differ in the number N of funds in each sample (written in the table's leftmost column), and across columns they differ in the number T of observations per fund in the sample (written in the table's top row). In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias; estimates with no bias have no shading and estimates with the maximum possible bias have the darkest shading.

	T = 180	T = 384	T = 500	T = 750	T = 1,000	T = 2,000
$N = 1,400$: $\pi^0 = 9.00\%$	68.10 (2.74)	45.16 (2.39)	36.23 (2.19)	23.59 (1.83)	16.95 (1.57)	9.67 (1.21)
$\pi^- = 78.00\%$	30.28 (2.40)	47.88 (2.14)	54.67 (2.00)	64.73 (1.75)	70.25 (1.59)	77.14 (1.37)
$\pi^+ = 13.00\%$	1.62 (1.04)	6.96 (1.01)	9.10 (1.00)	11.68 (0.99)	12.80 (0.99)	13.19 (0.98)
$N = 2,000$: $\pi^0 = 9.00\%$	68.34 (2.29)	45.14 (2.00)	36.14 (1.83)	23.60 (1.53)	16.89 (1.32)	9.61 (1.01)
$\pi^- = 78.00\%$	30.25 (2.01)	47.94 (1.79)	54.72 (1.67)	64.66 (1.47)	70.37 (1.33)	77.20 (1.14)
$\pi^+ = 13.00\%$	1.42 (0.87)	6.92 (0.84)	9.14 (0.84)	11.74 (0.83)	12.74 (0.83)	13.20 (0.82)
$N = 3,500$: $\pi^0 = 9.00\%$	68.31 (1.73)	44.91 (1.51)	36.11 (1.39)	23.66 (1.16)	16.95 (1.00)	9.62 (0.76)
$\pi^- = 78.00\%$	30.45 (1.52)	48.07 (1.35)	54.82 (1.26)	64.65 (1.11)	70.35 (1.00)	77.21 (0.87)
$\pi^+ = 13.00\%$	1.25 (0.66)	7.02 (0.64)	9.07 (0.63)	11.68 (0.63)	12.70 (0.62)	13.17 (0.62)
$N = 5,000$: $\pi^0 = 9.00\%$	68.29 (1.45)	45.02 (1.26)	36.20 (1.16)	23.73 (0.97)	17.06 (0.84)	9.68 (0.64)
$\pi^- = 78.00\%$	30.31 (1.27)	47.97 (1.13)	54.70 (1.06)	64.64 (0.93)	70.29 (0.84)	77.17 (0.73)
$\pi^+ = 13.00\%$	1.40 (0.55)	7.01 (0.53)	9.10 (0.53)	11.63 (0.52)	12.65 (0.52)	13.15 (0.52)

D. Additional Results

In this section, we present results from an additional simulation that is useful in understanding some of the main simulation results presented in the paper.

Table IA.VII is a variation of Table I in the paper, the difference being that in the former the number of observations per fund in each simulated sample is smaller. In more detail, in the simulations in both tables, nonzero α s are generated from a variety of discrete distributions ($\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+$), but in Table I in the paper the number of observations T per fund equals 384 while in Table IA.VII the number of observations T per fund equals 180. The value T=384 is the one that BSW use in their simulations, therefore we also use it for comparability in Tables I through IV in the paper. The value T=180 used in Table IA.VII is the mean number of observations per fund in the real data. We note that the samples of funds used in the simulations for both tables are balanced panels, so comparing the results in Tables I and IA.VII is useful in understanding whether the differences in the simulation results presented in Tables I and V in the paper are more due to using in the latter an unbalanced panel or more due to using a shorter panel.

Table IA.VII: Estimates of Alpha Group Proportions — Small T

Results from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions (i.e., $\alpha \sim \pi^0 \delta^0 + \pi^- \delta^-_{\bar{\alpha}} + \pi^+ \delta^+_{\bar{\alpha}}$ as in Table I), but the number of observations per fund is equal to 180, i.e., the mean number of observations per fund in the real data. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). In the table, we report the mean (across 1,000 repetitions) estimated proportions from the FDR methodology and their standard deviations (in parentheses). Each cell of the table is shaded, with darker shades corresponding to estimates with greater bias; estimates with no bias have no shading and estimates with the maximum possible bias have the darkest shading.

	$\bar{\alpha} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{\alpha}=2.5$	$\bar{\alpha} = 3.0$	$\bar{\alpha} = 3.5$
DGP <i>A-D-1</i> : $\pi^0 = 93.75\%$	98.33 (2.95)	97.82 (2.95)	97.21 (2.94)	96.76 (2.94)	95.94 (2.94)	95.07 (2.94)
$\pi^- = 5.75\%$	0.90 (2.62)	1.43 (2.62)	1.83 (2.62)	2.43 (2.61)	2.98 (2.61)	3.70 (2.61)
$\pi^{+} = 0.50\%$	0.78 (1.30)	0.75 (1.30)	0.96 (1.29)	0.82 (1.29)	1.08 (1.29)	1.22 (1.29)
DGP A-D-2: $\pi^0 = 87.50\%$	97.87 (2.95)	96.70 (2.94)	95.69 (2.94)	93.79 (2.93)	92.60 (2.93)	90.91 (2.92)
$\pi^- = 11.50\%$	1.47 (2.62)	2.51 (2.61)	3.71 (2.61)	5.21 (2.60)	6.34 (2.60)	7.87 (2.59)
$\pi^{+} = 1.00\%$	0.66 (1.29)	0.80 (1.28)	0.60 (1.28)	1.01 (1.27)	1.06 (1.27)	1.22 (1.26)
DGP <i>A-D-3</i> : $\pi^0 = 75.00\%$	96.84 (2.94)	94.45 (2.94)	91.21 (2.92)	88.05 (2.90)	84.70 (2.88)	81.67 (2.86)
$\pi^- = 23.00\%$	2.69 (2.60)	5.07 (2.59)	8.19 (2.58)	11.05 (2.57)	13.99 (2.55)	16.64 (2.54)
$\pi^{+} = 2.00\%$	0.47 (1.26)	0.48 (1.24)	0.60 (1.22)	0.90 (1.22)	1.31 (1.21)	1.69 (1.21)
DGP A-D-4: $\pi^0 = 62.50\%$	96.33 (2.94)	92.18 (2.92)	87.07 (2.90)	81.74 (2.86)	76.76 (2.82)	72.54 (2.78)
$\pi^- = 34.50\%$	3.44 (2.59)	7.46 (2.57)	12.40 (2.55)	17.26 (2.52)	21.58 (2.49)	25.22 (2.46)
$\pi^{+} = 3.00\%$	0.23 (1.23)	0.36 (1.20)	0.53 (1.18)	0.99 (1.17)	1.66 (1.15)	2.24 (1.15)
DGP A-D-5: $\pi^0 = 50.00\%$	94.65 (2.94)	89.60 (2.91)	82.60 (2.87)	75.72 (2.81)	68.84 (2.74)	63.39 (2.68)
$\pi^- = 46.00\%$	5.20 (2.58)	10.27 (2.55)	16.93 (2.51)	23.26 (2.47)	29.13 (2.42)	33.74 (2.37)
$\pi^{+} = 4.00\%$	0.15 (1.20)	0.13 (1.15)	0.47 (1.12)	1.02 (1.10)	2.04 (1.09)	2.87 (1.08)
DGP A-D-6: $\pi^0 = 37.50\%$	93.49 (2.93)	86.83 (2.90)	77.92 (2.83)	69.41 (2.75)	61.52 (2.66)	54.43 (2.55)
$\pi^- = 57.50\%$	6.45 (2.57)	13.07 (2.53)	21.71 (2.47)	29.54 (2.41)	36.28 (2.33)	42.15 (2.26)
$\pi^{+} = 5.00\%$	0.06 (1.16)	0.10 (1.11)	0.37 (1.07)	1.05 (1.04)	2.20 (1.02)	3.42 (1.01)
DGP A-D-7: $\pi^0 = 25.00\%$	92.09 (2.92)	84.05 (2.88)	74.14 (2.80)	63.28 (2.68)	53.71 (2.54)	45.13 (2.39)
$\pi^- = 69.00\%$	7.86 (2.55)	15.89 (2.50)	25.62 (2.43)	35.48 (2.34)	43.54 (2.23)	50.55 (2.11)
$\pi^{+} = 6.00\%$	0.05 (1.13)	0.06 (1.06)	0.25 (1.01)	1.24 (0.98)	2.75 (0.96)	4.32 (0.94)
DGP A-D-8: $\pi^0 = 6.25\%$	90.60 (2.92)	80.11 (2.85)	67.17 (2.73)	53.87 (2.54)	41.86 (2.32)	31.58 (2.07)
$\pi^- = 86.25\%$	9.40 (2.53)	19.88 (2.46)	32.69 (2.35)	44.75 (2.20)	54.48 (2.03)	63.07 (1.84)
$\pi^{+} = 7.50\%$	0.00 (1.07)	0.01 (0.98)	0.14 (0.92)	1.38 (0.88)	3.65 (0.84)	5.35 (0.82)

E. Interval Estimates

In this section, we present interval estimates corresponding to some of the simulations that we present in the paper. In particular:

- The results in Table IA.VIII correspond to the simulation results presented in Table I in the paper.
- The results in Table IA.IX correspond to the simulation results presented in Table V in the paper. In Tables IA.VIII and IA.IX we present, for each data generating process, the interval that contains 90% of the estimated proportions across 1,000 repetitions.

Table IA.VIII: Interval Estimates of Alpha Group Proportions — Baseline

Interval estimates corresponding to the simulations presented in Table I in the paper, i.e., simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions $(\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+)$. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). For each DGP, we conduct 1,000 simulations and calculate the estimated proportions from the FDR methodology. In the table, we report the interval that contains 90% of the estimated proportions across the 1,000 simulations. The results corresponding to the DGP used by BSW are enclosed in a border.

	$\bar{a} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{\alpha}=2.5$	$\bar{\alpha} = 3.0$	$\bar{\alpha} = 3.5$
DGP A-D-1: $\pi^0 = 93.75\%$	(93.4,99.9)	(92.9, 99.9)	(91.3, 99.9)	(90.7, 99.9)	(90.0, 99.6)	(89.8, 98.7)
$\pi^- = 5.75\%$	(0.0, 3.7)	(0.0, 4.6)	(0.0, 5.6)	(0.0, 6.5)	(0.4, 7.0)	(1.3, 7.2)
$\pi^{+} = 0.50\%$	(0.0, 4.0)	(0.0, 3.5)	(0.0, 3.8)	(0.0, 3.9)	(0.0, 3.9)	(0.0, 3.9)
DGP A-D-2: $\pi^0 = 87.50\%$	(91.8, 99.9)	(89.9, 99.7)	(87.5, 96.8)	(85.8, 95.3)	(84.6, 94.3)	(83.8, 92.5)
$\pi^- = 11.50\%$	(0.0, 5.0)	(0.3, 7.3)	(3.2, 9.3)	(4.7, 10.9)	(5.7, 11.9)	(7.5, 12.9)
$\pi^+ = 1.00\%$	(0.0, 4.2)	(0.0, 3.1)	(0.0, 4.2)	(0.0, 4.3)	(0.0, 4.6)	(0.0, 4.3)
DGP A-D-3: $\pi^0 = 75.00\%$	(90.4, 99.6)	(85.8, 94.5)	(80.2, 89.5)	(76.4, 85.6)	(73.8, 83.1)	(72.2, 80.6)
$\pi^- = 23.00\%$	(0.4, 8.6)	(5.5, 12.3)	(10.5, 16.6)	(14.4, 19.9)	(16.8, 22.3)	(19.2, 23.2)
$\pi^{+} = 2.00\%$	(0.0, 2.1)	(0.0, 2.9)	(0.0, 4.1)	(0.0, 4.6)	(0.0, 5.4)	(0.0, 5.3)
DGP A-D-4: $\pi^0 = 62.50\%$	(87.2,96.8)	(80.2, 90.1)	(72.9, 81.0)	(66.3,75.3)	(62.9, 71.4)	(60.1, 67.7)
$\pi^- = 34.50\%$	(3.3, 11.2)	(9.9, 17.5)	(18.5, 23.3)	(24.0, 28.8)	(27.8, 32.0)	(30.5, 33.9)
$\pi^+ = 3.00\%$	(0.0, 2.1)	(0.0, 3.5)	(0.0, 4.9)	(0.0, 5.8)	(0.0, 6.2)	(0.6, 6.4)
DGP A-D-5: $\pi^0 = 50.00\%$	(85.2, 95.2)	(74.6, 84.4)	(66.4,74.3)	(57.4,66.0)	(52.9, 60.5)	(49.2, 56.8)
$\pi^- = 46.00\%$	(4.8, 13.7)	(15.6, 22.9)	(25.3, 30.7)	(33.0, 37.2)	(38.2, 42.0)	(41.2, 44.6)
$\pi^{+} = 4.00\%$	(0.0, 1.1)	(0.0, 3.1)	(0.0, 4.5)	(0.0, 6.3)	(0.5, 6.5)	(1.6, 7.0)
DGP <i>A-D-6</i> : $\pi^0 = 37.50\%$	(81.5,91.1)	(69.6, 79.4)	(58.2, 66.7)	(48.1, 56.6)	(41.4, 49.2)	(37.6, 44.4)
$\pi^- = 57.50\%$	(8.9, 17.6)	(20.6, 28.3)	(32.5, 38.2)	(41.6, 46.1)	(47.9, 52.1)	(51.9, 55.4)
$\pi^{+} = 5.00\%$	(0.0, 0.7)	(0.0, 3.1)	(0.0, 5.0)	(0.9, 6.5)	(2.3, 7.7)	(3.1, 7.9)
DGP A-D-7: $\pi^0 = 25.00\%$	(79.8, 89.6)	(64.9,74.2)	(49.9, 58.7)	(39.4, 46.4)	(31.3, 36.8)	(26.7, 32.1)
$\pi^- = 69.00\%$	(10.4, 19.9)	(25.8, 33.4)	(40.3, 45.3)	(50.0, 54.8)	(58.1, 61.6)	(62.5,65.8)
$\pi^{+} = 6.00\%$	(0.0, 0.4)	(0.0, 2.6)	(0.0, 5.1)	(2.7, 6.8)	(4.1, 8.4)	(4.4, 8.2)
DGP A-D-8: $\pi^0 = 6.25\%$	(76.0, 85.2)	(57.1,67.5)	(39.3, 48.3)	(24.4, 32.5)	(14.9, 20.6)	(9.7, 14.0)
$\pi^- = 86.25\%$	(14.8, 24.0)	(32.5, 42.2)	(50.1, 56.1)	(63.2, 68.5)	(73.0, 76.6)	(79.1, 82.0)
$\pi^{+} = 7.50\%$	(0.0, 0.0)	(0.0, 2.0)	(0.7, 5.8)	(3.8, 7.9)	(5.8, 9.2)	(6.6, 9.2)

Table IA.IX: Interval Estimates of Alpha Group Proportions — Unbalanced Panel

Interval estimates corresponding to the simulations presented in Table V in the paper, i.e., simulations in which the sample of funds is an unbalanced panel and nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions ($\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{a}}^- + \pi^+ \delta_{\bar{a}}^+$). Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance \bar{a} of nonzero αs from zero (written in the table's top row). For each DGP, we conduct 1,000 simulations and calculate the estimated proportions from the FDR methodology. In the table, we report the interval that contains 90% of the estimated proportions across the 1,000 simulations.

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\pi^- = 5.75\%$ (0.0, 3.0) (0.0, 3.0) (0.0, 3.9) (0.0, 5.0) (0.0, 5.4)) (92.3,99.9)
) (0.0, 5.9)
$\pi^+ = 0.50\%$ (0.0, 2.5) (0.0, 2.4) (0.0, 3.6) (0.0, 3.4) (0.0, 3.2)) (0.0, 2.8)
DGP A-D-2: $\pi^0 = 87.50\%$ (94.7,99.9) (93.1,99.9) (91.6,99.2) (89.1,97.8) (88.4,96.3)	(87.4, 95.4)
$\pi^- = 11.50\%$ (0.0, 3.4) (0.0, 5.0) (0.8, 6.4) (2.2, 7.8) (3.7, 8.7)) (4.6, 9.7)
$\pi^{+} = 1.00\%$ (0.0, 2.1) (0.0, 2.5) (0.0, 2.6) (0.0, 3.6) (0.0, 3.4)) (0.0, 3.4)
DGP A-D-3: $\pi^0 = 75.00\%$ (93.1,99.9) (90.6,98.8) (87.7,95.7) (84.3,91.8) (81.5,89.2)	(78.8, 86.8)
$\pi^- = 23.00\%$ (0.0, 5.6) (1.2, 7.4) (4.3,11.2) (8.0,13.5) (10.8,15.9)) (13.1, 17.6)
$\pi^+ = 2.00\%$ (0.0, 1.8) (0.0, 2.0) (0.0, 2.2) (0.0, 3.4) (0.0, 3.6)) (0.0, 4.3)
DGP A-D-4: $\pi^0 = 62.50\%$ (92.3,99.9) (87.4,96.7) (83.4,91.7) (78.4,86.0) (74.2,81.7)) (70.2,77.8)
$\pi^- = 34.50\%$ (0.0, 7.4) (3.3,11.2) (8.3,15.2) (14.0,19.1) (18.0,22.7)) (21.7, 26.2)
$\pi^{+}=3.00\%$ (0.0, 1.2) (0.0, 1.5) (0.0, 2.1) (0.0, 3.5) (0.0, 3.8)	(0.0, 4.6)
DGP A-D-5: $\pi^0 = 50.00\%$ (91.3, 98.9) (85.2, 92.9) (79.1, 87.1) (72.6, 80.3) (67.1, 74.7)) (62.2, 69.3)
$\pi^- = 46.00\%$ (1.1, 8.6) (7.0, 13.8) (12.9, 19.9) (19.7, 25.6) (25.3, 29.9)) (29.7, 34.1)
$\pi^+ = 4.00\%$ (0.0, 0.4) (0.0, 1.1) (0.0, 1.7) (0.0, 3.1) (0.0, 3.8)) (0.1, 4.6)
DGP A-D-6: $\pi^0 = 37.50\%$ (89.1, 98.0) (82.7, 91.1) (75.2, 82.9) (67.1, 74.9) (59.4, 67.4)) (53.8,61.1)
$\pi^- = 57.50\%$ (2.0, 10.5) (8.9, 16.9) (17.1, 24.0) (25.1, 31.0) (32.4, 36.9)) (37.9, 42.2)
$\pi^{+} = 5.00\%$ (0.0, 0.2) (0.0, 0.6) (0.0, 1.5) (0.0, 2.7) (0.0, 4.3)) (0.5, 5.1)
DGP A-D-7: $\pi^0 = 25.00\%$ (88.1, 96.2) (80.1, 88.4) (70.7, 79.3) (61.2, 68.7) (52.3, 59.7)) (45.2,52.3)
$\pi^{-} = 69.00\%$ (3.8,11.7) (11.6,19.8) (20.7,28.6) (31.2,36.7) (39.4,44.2)) (45.3,50.1)
$\pi^{+} = 6.00\%$ (0.0, 0.0) (0.0, 0.1) (0.0, 1.1) (0.0, 2.9) (0.2, 4.8)) (1.8, 5.8)
DGP A-D-8: $\pi^0 = 6.25\%$ (86.5, 94.9) (76.0, 84.0) (64.9, 72.4) (51.5, 59.6) (41.7, 48.8)) (32.2, 38.9)
$\pi^{-} = 86.25\%$ (5.1,13.5) (16.0,24.0) (27.6,34.7) (40.1,46.0) (49.9,54.7)) (57.6,62.3)
$\pi^+ = 7.50\%$ (0.0, 0.0) (0.0, 0.0) (0.0, 0.6) (0.0, 2.9) (0.4, 4.7)) (2.3, 6.5)

F. Coverage Probability of Confidence Intervals

In this section, we present results on the coverage probabilities of the confidence intervals corresponding to the simulation results presented in Table I in the paper. That is we present, for each data generating process, the percentage (across 1,000 repetitions) of samples such that the true proportion is contained in the confidence interval constructed by the FDR methodology. For example, to calculate the coverage probability of the 90% confidence interval for π^0 , we calculate the proportion of samples such that the true value of π^0 is contained in the interval $\hat{\pi}^0 \pm 1.645 \cdot \hat{\sigma}_{\hat{\pi}^0}$, where $\hat{\pi}^0$ is the FDR estimate of π^0 and $\hat{\sigma}_{\hat{\pi}^0}$ is the analytic standard deviation of this estimate. In Tables IA.X, IA.XI, and IA.XII we present, respectively, the coverage probabilities for the 90%, 95%, and 99% confidence intervals constructed by the FDR methodology.

Table IA.X: Proportion of 90% Confidence Intervals Containing True Proportions of Alpha Groups

Results corresponding to the simulations presented in Table I in the paper, i.e., from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions $(\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+)$. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). In the table, we report the percentage (across 1,000 repetitions) of samples such that the true proportion is contained in the 90% confidence interval constructed by the FDR methodology (e.g., for π^0 , it is $\hat{\pi}^0 \pm 1.645 \cdot \hat{\sigma}_{\hat{\pi}^0}$). Each cell of the table is shaded, with darker (lighter) shades corresponding to cases in which the *actual* percentage of confidence intervals containing the true value is farther (closer) to the *nominal* coverage of 90%. The results corresponding to the DGP used by BSW are enclosed in a border.

		$\bar{a} = 1.0$	$\bar{a} = 1.5$	$\bar{a} = 2.0$	$\bar{a}=2.5$	$\bar{a} = 3.0$	$\bar{a} = 3.5$
DGP <i>A-D-</i> 1:	$\pi^0 = 93.75\%$	49.50%	65.50%	79.00%	85.00%	89.00%	91.50%
	$\pi^- = 5.75\%$	38.00%	59.00%	77.50%	86.00%	90.00%	94.00%
	$\pi^{+} = 0.50\%$	87.50%	90.50%	84.00%	83.00%	86.00%	85.00%
DGP <i>A-D-</i> 2:	$\pi^0 = 87.50\%$	8.00%	18.00%	52.50%	76.00%	85.50%	92.50%
	$\pi^- = 11.50\%$	0.00%	5.50%	45.00%	78.00%	89.00%	97.00%
	$\pi^+ = 1.00\%$	92.00%	95.50%	88.50%	85.00%	83.50%	88.50%
DGP <i>A-D-</i> 3:	$\pi^0 = 75.00\%$	0.00%	0.50%	4.50%	33.50%	70.00%	90.50%
	$\pi^- = 23.00\%$	0.00%	0.00%	0.00%	16.50%	75.00%	96.50%
	$\pi^+ = 2.00\%$	83.00%	69.50%	66.00%	74.00%	71.00%	69.00%
DGP <i>A-D-</i> 4:	$\pi^0 = 62.50\%$	0.00%	0.00%	0.00%	7.50%	48.50%	87.50%
	$\pi^- = 34.50\%$	0.00%	0.00%	0.00%	0.00%	30.50%	94.50%
	$\pi^+ = 3.00\%$	14.50%	23.50%	56.50%	62.50%	65.50%	74.00%
DGP <i>A-D-</i> 5:	$\pi^0 = 50.00\%$	0.00%	0.00%	0.00%	0.00%	17.50%	75.00%
	$\pi^- = 46.00\%$	0.00%	0.00%	0.00%	0.00%	3.50%	72.00%
	$\pi^+ = 4.00\%$	2.00%	11.00%	34.50%	61.50%	62.50%	75.00%
DGP <i>A-D-</i> 6:	$\pi^0 = 37.50\%$	0.00%	0.00%	0.00%	0.00%	4.50%	59.50%
	$\pi^- = 57.50\%$	0.00%	0.00%	0.00%	0.00%	0.00%	33.50%
	$\pi^+ = 5.00\%$	0.50%	5.00%	25.00%	53.50%	74.00%	69.50%
DGP <i>A-D-7</i> :	$\pi^0 = 25.00\%$	0.00%	0.00%	0.00%	0.00%	0.00%	25.50%
	$\pi^- = 69.00\%$	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%
	$\pi^+ = 6.00\%$	0.00%	0.50%	13.00%	58.50%	79.00%	76.50%
DGP <i>A-D-</i> 8:	$\pi^0 = 6.25\%$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	$\pi^- = 86.25\%$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	$\pi^+ = 7.50\%$	0.00%	0.00%	4.00%	43.50%	84.50%	85.50%

Table IA.XI: Proportion of 95% Confidence Intervals Containing True Proportions of Alpha Groups

Results corresponding to the simulations presented in Table I in the paper, i.e., from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions $(\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+)$. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). In the table, we report the percentage (across 1,000 repetitions) of samples such that the true proportion is contained in the 95% confidence interval constructed by the FDR methodology (e.g., for π^0 , it is $\hat{\pi}^0 \pm 1.960 \cdot \hat{\sigma}_{\hat{\pi}^0}$). Each cell of the table is shaded, with darker (lighter) shades corresponding to cases in which the *actual* percentage of confidence intervals containing the true value is farther (closer) to the *nominal* coverage of 95%. The results corresponding to the DGP used by BSW are enclosed in a border.

aabaa 5	$\bar{a} = 3.5$
% 95.00%	
	98.00%
% 94.50%	97.50%
% 88.50%	89.50%
% 92.00%	99.00%
% 92.50%	99.00%
% 87.00%	90.00%
% 79.50%	94.00%
% 85.50%	97.50%
% 91.50%	87.50%
% 62.50%	95.00%
% 53.00%	97.50%
% 74.50%	78.00%
% 27.00%	85.00%
% 8.50%	89.50%
% 71.00%	85.00%
% 8.00%	72.00%
% 0.00%	67.00%
% 79.00%	80.00%
% 0.00%	37.50%
% 0.00%	12.00%
% 85.00%	86.00%
% 0.00%	0.50%
% 0.00%	0.00%
% 89.00%	93.00%
	% 88.50% % 92.00% % 92.50% % 87.00% % 79.50% % 85.50% % 53.00% % 27.00% % 8.50% % 71.00% % 0.00% % 0.00% % 0.00% % 85.00% % 0.00% % 0.00% % 0.00% % 0.00% % 0.00% % 0.00% % 0.00% % 0.00%

Table IA.XII: Proportion of 99% Confidence Intervals Containing True Proportions of Alpha Groups

Results corresponding to the simulations presented in Table I in the paper, i.e., from simulations in which nonzero αs (expressed as annualized percentages) are generated from a variety of discrete distributions $(\alpha \sim \pi^0 \delta^0 + \pi^- \delta_{\bar{\alpha}}^- + \pi^+ \delta_{\bar{\alpha}}^+)$. Across rows, the DGPs differ in the true proportions π^0 , π^- , and π^+ (written in the table's leftmost column), and across columns they differ in the distance $\bar{\alpha}$ of nonzero αs from zero (written in the table's top row). In the table, we report the percentage (across 1,000 repetitions) of samples such that the true proportion is contained in the 99% confidence interval constructed by the FDR methodology (e.g., for π^0 , it is $\hat{\pi}^0 \pm 2.575 \cdot \hat{\sigma}_{\hat{\pi}^0}$). Each cell of the table is shaded, with darker (lighter) shades corresponding to cases in which the *actual* percentage of confidence intervals containing the true value is farther (closer) to the *nominal* coverage of 99%. The results corresponding to the DGP used by BSW are enclosed in a border.

		$\bar{\alpha} = 1.0$	$\bar{\alpha} = 1.5$	$\bar{\alpha} = 2.0$	$\bar{\alpha} = 2.5$	$\bar{\alpha} = 3.0$	$\bar{\alpha} = 3.5$
DGP <i>A-D-</i> 1:	$\pi^0 = 93.75\%$	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	$\pi^- = 5.75\%$	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	$\pi^{+} = 0.50\%$	94.50%	96.50%	95.50%	93.50%	94.50%	95.00%
DGP <i>A-D-2</i> :	$\pi^0 = 87.50\%$	24.50%	53.00%	83.00%	94.50%	97.50%	100.00%
	$\pi^- = 11.50\%$	7.00%	47.50%	83.00%	94.50%	98.50%	100.00%
	$\pi^+ = 1.00\%$	96.00%	98.00%	95.50%	94.00%	93.50%	95.00%
DGP <i>A-D-</i> 3:	$\pi^0 = 75.00\%$	0.00%	1.00%	20.50%	73.00%	93.00%	98.50%
	$\pi^- = 23.00\%$	0.00%	0.00%	7.00%	71.50%	96.00%	99.50%
	$\pi^+ = 2.00\%$	100.00%	99.00%	99.00%	97.50%	94.00%	93.50%
DGP <i>A-D-</i> 4:	$\pi^0 = 62.50\%$	0.00%	0.00%	0.00%	31.00%	84.00%	98.00%
	$\pi^- = 34.50\%$	0.00%	0.00%	0.00%	11.50%	89.00%	100.00%
	$\pi^+ = 3.00\%$	92.50%	67.50%	80.50%	87.50%	86.00%	89.50%
DGP <i>A-D-5</i> :	$\pi^0 = 50.00\%$	0.00%	0.00%	0.00%	3.50%	56.00%	95.00%
	$\pi^- = 46.00\%$	0.00%	0.00%	0.00%	0.00%	39.50%	99.50%
	$\pi^+ = 4.00\%$	6.00%	24.00%	62.00%	80.00%	89.50%	92.00%
DGP <i>A-D-</i> 6:	$\pi^0 = 37.50\%$	0.00%	0.00%	0.00%	0.00%	20.50%	91.00%
	$\pi^- = 57.50\%$	0.00%	0.00%	0.00%	0.00%	4.50%	93.00%
	$\pi^+ = 5.00\%$	0.50%	9.00%	44.00%	78.50%	87.50%	91.00%
DGP <i>A-D-</i> 7:	$\pi^0 = 25.00\%$	0.00%	0.00%	0.00%	0.00%	0.50%	68.00%
	$\pi^- = 69.00\%$	0.00%	0.00%	0.00%	0.00%	0.00%	52.00%
	$\pi^{+} = 6.00\%$	0.00%	2.00%	29.50%	81.50%	94.50%	96.00%
DGP <i>A-D-</i> 8:	$\pi^0 = 6.25\%$	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%
	$\pi^- = 86.25\%$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	$\pi^+ = 7.50\%$	0.00%	0.00%	11.00%	69.50%	95.00%	100.00%

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